MATHEMATICS 9th Science Group

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Chapter 5.

Factorization

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Contents

Introduction:

Factorization plays an important role in mathematics as it helps to reduce the study of complicated expressions to the study of simpler expressions. In this unit, we will deal different types of factorization of polynimials.

Factorization:

If a polynomial $p(x)$ can be expressed as $p(x) = g(x)h(x)$, then each of the polynomial $g(x) and\ h(x)$ is called a factor of $f(x)$.

Factorization of the expression of the type $ka + kb + kc$.

We will take common k from every term of the expression

$$ka + kb + kc = k(a + b + c)$$

Factorization of the type $ac + ad + bc + bd$

$$ac + ad + bc + bd = a(c + d) + b(c + d)$$
$$= (c + d)(a + b)$$

Factorization of the type $a^2 \pm 2ab + b^2$

$$(i).\quad a^2 + 2ab + b^2 = (a + b)^2$$
$$= (a + b)(a + b)$$
$$(ii).\quad a^2 - 2ab + b^2 = (a - b)^2$$
$$= (a - b)(a - b)$$

Factorization of the type $a^2 - b^2$

$$a^2 - b^2 = (a + b)(a - b)$$

Factorization of the type $a^2 \pm 2ab + b^2 - c^2$

$$a^2 \pm 2ab + b^2 - c^2 = (a \pm b)^2 - c^2$$
$$= (a \pm b + c)(a \pm b - c)$$

Exercise 5.1

Factorize.Question.1.

(i). $2abc - 4abx + 2abd$

Solution.

$$2abc - 4abx + 2abd = 2ab(c - 2x + d)$$

Answer.

(ii). $9xy - 12x^2y + 18y^2$

Solution.

$$9xy - 12x^2y + 18y^2 = 3y(3x - 4x^2 + 6y)$$

Answer.

(iii). $-3x^2y - 3x + 9xy^2$

Solution.

$$-3x^2y - 3x + 9xy^2 = -3x(y + 1 - 3y^2)$$

Answer.

(iv). $5ab^2c^3 - 10a^2b^3c - 20a^3bc^2$

Solution.

$$5ab^2c^3 - 10a^2b^3c - 20a^3bc^2 = 5abc(bc^2 - 2ab^2 - 4a^2c)$$

Answer.

(v). $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

Solution.

$$3x^3\, y(x - 3y) - 7x^2\, y^2\, (x - 3y)$$
$$= x^2y(x - 3y)(3x - 7y)$$

Answer.

(vi). $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

Solution.

$$2xy^3(x^2 + 5) + 8xy^2(x^2 + 5) = 2xy^2(x^2 + 5)(y + 4)$$

Answer.

Question.2.

(i). $5ax - 3ay - 5bx + 3by$

Solution.

$$5ax - 3ay - 5bx + 3by$$
$$= a(5x - 3y) - b(5x - 3y)$$
$$= (5x - 3y)(a - b)$$

Answer.

(ii). $3xy + 2y - 12x - 8$

Solution.

$$3xy + 2y - 12x - 8 = y(3x + 2) - 4(3x + 2)$$
$$= (3x + 2)(y - 4)$$

Answer.

(iii). $x^3 + 3xy^2 - 2x^2y - 6y^3$

Solution.

$$x^3 + 3xy^2 - 2x^2y - 6y^3$$
$$= x(x^2 + 3y^2) - 2y(x^2 + 3y^2)$$
$$= (x^2 + 3y^2)(x - 2y)$$

Answer.

(iv). $(x^2 - y^2)z + (y^2 - z^2)x$

Solution.

$$(x^2 - y^2)z + (y^2 - z^2)x$$
$$= x^2z - y^2z + y^2x - z^2x$$
$$= x^2z + y^2x - y^2z - z^2x$$
$$= x(xz + y^2) - z(y^2 + xz)$$
$$= (xz + y^2)(x - z)$$

Answer.

Question.3.

(i). $144a^2 + 24a + 1$

Solution.

$$144a^2 + 24a + 1 = (12a)^2 + 2(12a)(1) + (1)^2$$
$$= (12a + 1)^2$$

(ii). $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

Solution.

$$\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2} = \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$$
$$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

(iii). $(x+y)^2 - 14z(x+y) + 49z^2$

Solution.

$$(x+y)^2 - 14z(x+y) + 49z^2 = (x+y)^2 - 2(x+y)(7z) + (7z)^2$$
$$= (x+y-7z)^2$$

(iv). $12x^2 - 36x + 27$

Solution.

$$12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$$
$$= 3[(2x)^2 - 2(2x)(3) + (3)^2]$$
$$= 3(2x-3)^2$$

Question.4.

(i). $3x^2 - 75y^2$

Solution.

$$3x^2 - 75y^2 = 3(x^2 - 25y^2)$$
$$= 3[(x)^2 - (5y)^2]$$
$$= 3(x+5y)(x-5y)$$

(ii). $x(x-1) - y(y-1)$

Solution.

$$x(x-1) - y(y-1) = x^2 - x - y^2 + y$$
$$= x^2 - y^2 - x + y$$
$$= (x+y)(x-y) - 1(x-y)$$
$$= (x-y)(x+y-1)$$

(iii). $128am^2 - 242an^2$

Solution.

$$128am^2 - 242an^2 = 2a(64m^2 - 121n^2)$$
$$= 2a[(8m)^2 - (11n)^2]$$
$$= 2a(8m+11n)(8m-11n)$$

(iv). $3x - 243x^3$

Solution.

$$3x - 243x^3 = 3x(1 - 81x^2)$$
$$= 3x[(1)^2 - (9x)^2]$$
$$= 3x(1+9x)(1-9x)$$

Question.5.

(i). $x^2 - y^2 - 6y - 9$

Solution.

$$x^2 - y^2 - 6y - 9 = x^2 - (y^2 + 6y + 9)$$
$$= x^2 - [(y)^2 + 2(y)(3) + (3)^2]$$
$$= x^2 - (y+3)^2$$
$$= [x + (y+3)][x - (y+3)]$$
$$= (x+y+3)(x-y-3)$$

(ii). $x^2 - a^2 + 2a - 1$

Solution.

$$x^2 - a^2 + 2a - 1 = x^2 - (a^2 - 2a + 1)$$
$$= x^2 - [(a)^2 - 2(a)(1) + (1)^2]$$
$$= (x)^2 - (a-1)^2$$
$$= [x + (a-1)][x - (a-1)]$$
$$= (x+a-1)(x-a+1)$$

(iii). $4x^2 - y^2 - 2y - 1$

Solution.

$$4x^2 - y^2 - 2y - 1 = 4x^2 - (y^2 + 2y + 1)$$
$$= 4x^2 - [(y)^2 + 2(y)(1) + (1)^2]$$
$$= (2x)^2 - (y+1)^2$$
$$= [2x + (y+1)][2x - (y+1)]$$
$$= (2x+y+1)(2x-y-1)$$

(iv). $x^2 - y^2 - 4x - 2y + 3$

Solution.

$$x^2 - y^2 - 4x - 2y + 3 = x^2 - 4x - y^2 - 2y + 3$$
$$= x^2 - 4x + 4 - y^2 - 2y - 1$$
$$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$$
$$= [(x)^2 - 2(x)(2) + (2)^2] - [(y)^2 + 2(y)(1) + (1)^2]$$
$$= (x-2)^2 - (y+1)^2$$
$$= [(x-2) + (y+1)][(x-2) - (y+1)]$$
$$= (x-2+y+1)(x-2-y-1)$$
$$= (x+y-1)(x-y-3)$$

(v). $25x^2 - 10x + 1 - 36z^2$

Solution.

$$25x^2 - 10x + 1 - 36z^2 = [(5x)^2 - 2(5x)(1) + (1)^2] - (6z)^2$$
$$= (5x-1)^2 - (6z)^2$$
$$= (5x-1+6z)(5x-1-5z)$$
$$= (5x+6z-1)(5x-5z-1)$$

(vi). $x^2 - y^2 - 4xz + 4z^2$

Solution.

$$x^2 - y^2 - 4xz + 4z^2 = x^2 - 4xz + 4z^2 - y^2$$
$$= (x)^2 - 2(x)(z) + (2z)^2 - y^2$$
$$= (x-2z)^2 - (y)^2$$
$$= (x-2z+y)(x-2z-y)$$
$$= (x+y-2z)(x-y-2z)$$

(a) **Factorization of the Expression of the types:**

$a^4+a^2b^2+b^4$ or a^4+4b^4

Explanation: For $a^4+a^2b^2+b^4$

$$a^4+a^2b^2+b^4=a^4+b^4+a^2b^2$$
$$=(a^2)^2+(b^2)^2+2(a^2)(b^2)-2(a^2)(b^2)+a^2b^2$$
$$=(a^2+b^2)^2-2a^2b^2+a^2b^2$$
$$=(a^2+b^2)^2-a^2b^2$$
$$=(a^2+b^2)^2-(ab)^2$$
$$=(a^2+b^2+ab)(a^2+b^2-ab)$$

Explanation: For a^4+4b^4

$$a^4+4b^4=(a^2)^2+(2b^2)^2+2(a^2)(2b^2)-2(a^2)(2b^2)$$
$$=(a^2+2b^2)^2-4a^2b^2$$
$$=(a^2+2b^2)^2-(2ab)^2$$
$$=(a^2+2b^2+2ab)(a^2+2b^2-2ab)$$

(b) **Factorization of the Expression of the types:**

x^2+px+q

Explanation:

$$x^2+px+q=x^2+(s+r)x+q\ ,\quad where\ s+r=p\ and\ s\times r=q$$
$$=x^2+sx+rx+s\times r$$
$$=x(x+s)+r(x+s)$$
$$=(x+s)(x+r)$$

(c) **Factorization of the Expression of the types:**

$$ax^2+bx+c,a\neq 0$$
$$ax^2+bx+c=ax^2+(s+r)x+c,$$
$$s+r=b\ and\ s\times r=ac$$
$$=ax^2+sx+rx+\frac{s\times r}{a}$$
$$=x(ax+s)+r\left(x+\frac{s}{a}\right)$$
$$=x(ax+s)+r\left(\frac{ax+s}{a}\right)$$
$$=x(ax+s)+\frac{r}{a}(ax+s)$$
$$=(ax+s)\left(x+\frac{r}{a}\right)$$

(d) **Factorization of the Expression of the types:**

(i). $(ax^2+bx+c)(ax^2+bx+d)+k$

(ii). $(x+a)(x+b)(x+c)(x+d)+k$

(iii). $(x+a)(x+b)(x+c)(x+d)+kx^2$

Explanation: For $(ax^2+bx+c)(ax^2+bx+d)+k$

$$(ax^2+bx+c)(ax^2+bx+d)+k$$

We will suppose $ax^2+bx=y$, *then above becomes*

$$=(y+c)(y+d)+k$$
$$=y^2+yd+yc+k$$
$$=y^2+(d+c)y+k$$

This the same type (b).

Explanation: For $(x+a)(x+b)(x+c)(x+d)+k$

$$(x+a)(x+b)(x+c)(x+d)+k$$

We will multiply the pair for which $a+b=c+d$, then

$$=[(x+a)(x+b)][(x+c)(x+d)]+k$$
$$=[x^2+bx+ax+ab][x^2+dx+cx+cd]+k$$
$$=(x^2+(b+a)x+ab)(x^2+(d+c)x+cd)+k$$

As $a+b=c+d$, *then*

$$=[x^2+(c+d)x+ab][x^2+(c+d)x+cd]+k$$

Suppose that

$x^2+(c+d)x=y$, *then above expression becomes*

$$=(y+ab)(y+cd)+k$$
$$=y^2+ycd+yab+abcd+k$$
$$=y^2+(cd+ab)y+abcd+k$$

This the same type (b).

Explanation: For $(ax^2+bx+c)(ax^2+bx+d)+kx^2$

$$(x+a)(x+b)(x+c)(x+d)+kx^2$$

We will multiply the pair for which $a+b=c+d$, then

$$=[(x+a)(x+b)][(x+c)(x+d)]+kx^2$$
$$=[x^2+bx+ax+ab][x^2+dx+cx+cd]+kx^2$$
$$=(x^2+(b+a)x+ab)(x^2+(d+c)x+cd)+kx^2$$

As $a+b=c+d$, *then*

$$=[x^2+(c+d)x+ab][x^2+(c+d)x+cd]+kx^2$$

Suppose that

$x^2+(c+d)x=y$, *then above expression becomes*

$$=(y+ab)(y+cd)+kx^2$$
$$=y^2+ycd+yab+abcd+kx^2$$
$$=y^2+(cd+ab)y+abcd+kx^2$$

After simplification it also becomes type (b).

(e). Factorization of the Expression of the type:

(i). $a^3+3a^2b+3ab^2+b^3$

(ii). $a^3-3a^2b+3ab^2-b^3$

Explanation For $a^3+3a^2b+3ab^2+b^3$

$$a^3+3a^2b+3ab^2+b^3=(a+b)^3$$

It's a very famous formula.

Explanation For $a^3-3a^2b+3ab^2-b^3$

$$a^3-3a^2b+3ab^2-b^3=(a-b)^3$$

It's a very famous formula.

(f). Factorization of the Expression of the type:

(i).a^3+b^3

(ii). a^3-b^3

We will use, well known formulas for these types

(i). $a^3+b^3=(a+b)(a^2-ab+b^2)$

(ii). $a^3-b^3=(a-b)(a^2+ab+b^2)$

Exercise 5.2

Factorize

Question.1.

(i). $x^4+\frac{1}{x^4}-3$

Solution.

$$x^4+\frac{1}{x^4}-3=(x^2)^2+\left(\frac{1}{x^2}\right)^2-2(x^2)\left(\frac{1}{x^2}\right)+2(x^2)\left(\frac{1}{x^2}\right)-3$$
$$=\left(x^2-\frac{1}{x^2}\right)^2+2-3$$
$$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$$
$$=(x^2-\frac{1}{x^2}+1)(x^2-\frac{1}{x^2}-1)$$

(ii). $3x^4+12y^4$

Solution.

$$3x^4+12y^4=3(x^4+4y^4)$$
$$=3[(x^2)^2+(2y^2)^2+2(x^2)(2y^2)-2(x^2)(2y^2)]$$
$$=3[(x^2+2y^2)^2-4x^2y^2]$$
$$=3[(x^2+2y^2)^2-(2xy)^2]$$
$$=3(x^2+2y^2+2xy)(x^2+2y^2-2xy)$$
$$=3(x^2+2xy+2y^2)(x^2-2xy+2y^2)$$

(iii). $a^4++3a^2b^2+4b^4$

Solution.

$$a^4++3a^2b^2+4b^4=a^4+4b^4+3a^2b^2$$
$$=(a^2)^2+(2b^2)^2+2(a^2)(2b^2)-2(a^2)(2b^2)+3a^2\,b^2$$
$$=(a^2+2b^2)^2-4a^2b^2+3a^2b^2$$
$$=(a^2+2b^2)^2-a^2b^2$$
$$=(a^2+2b^2)^2-(ab)^2$$
$$=(a^2+2b^2+ab)(a^2+2b^2-ab)$$

(iv). $4x^4+81$

Solution.

$$4x^4+81=(2x^2)^2+(9)^2+2(2x^2)(9)-2(2x^2)(9)$$
$$=(2x^2+9)^2-36x^2$$
$$=(2x^2+9)^2-(6x)^2$$
$$=(2x^2+9+6x)(2x^2+9-6x)$$
$$=(2x^2+6x+9)(2x^2-6x+9)$$

(v). x^4+x^2+25

Solution.

$$x^4+x^2+25=x^4+25+x^2$$
$$=(x^2)^2+(5)^2+2(x^2)(5)-2(x^2)(5)+x^2$$
$$=(x^2+5)^2-10x^2+x^2$$
$$=(x^2+5)^2-9x^2$$
$$=(x^2+5)^2-(3x)^2$$
$$=(x^2+5+3x)(x^2+5-3x)$$
$$=(x^2+3x+5)(x^2-3x+5)$$

(v). x^4+4x^2+16

Solution.

$$x^4+4x^2+16=x^4+16+4x^2$$
$$=(x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$$
$$=(x^2+4)^2-8x^2+4x^2$$
$$=(x^2+4)^2-4x^2$$
$$=(x^2+4)^2-(2x)^2$$
$$=(x^2+4+2x)(x^2+4-2x)$$
$$=(x^2+2x+4)(x^2-2x+4)$$

Question.2.

(i). $x^2+14x+48$

Solution.

$$x^2+14x+48=x^2+8x+6x+48$$
$$=x(x+8)+6(x+8)$$
$$=(x+8)(x+6)$$

(ii). $x^2-21x+108$

Solution.

$$x^2-21x+108=x^2-12x-9x+108$$
$$=x(x-12)-9(x-12)$$
$$=(x-12)(x-9)$$

(iii). $x^2-11x-42$

Solution.

$$x^2-11x-42=x^2-14x+3x-42$$
$$=x(x-14)+3(x-14)$$
$$=(x-14)(x+3)$$

(iii). $x^2+x-132$

Solution.

$$x^2+x-132=x^2+12x-11x-132$$
$$=x(x+12)-11(x+12)$$
$$=(x+12)(x-11)$$

Question.3.

(i). $4x^2+12x+5$

Solution.

$$4x^2+12x+5=4x^2+10x+2x+5$$
$$=2x(2x+5)+1(2x+5)$$
$$=(2x+5)(2x+1)$$

(ii). $30x^2+7x-15$

Solution.

$$30x^2+7x-15=30x^2+25x-18x-15$$
$$=5x(6x+5)-3(6x+5)$$
$$=(6x+5)(5x-3)$$

(iii). $24x^2-65x+21$

Solution.

$$24x^2-65x+21=24x^2-56x-9x+21$$
$$=8x(3x-7)-3(3x-7)$$
$$=(3x-7)(8x-3)$$

(iv). $5x^2-16x-21$

Solution.

$$5x^2-16x-21=5x^2-21x+5x-21$$
$$=x(5x-21)+1(5x-21)$$
$$=(5x-21)(x+1)$$

(v). $4x^2-17xy+4y^2$

Solution.

$$4x^2-17xy+4y^2=4x^2-16xy-xy+4y^2$$
$$=4x(x-4y)-y(x-4y)$$
$$=(x-4y)(4x-y)$$

(vi). $3x^2-38xy-13y^2$

Solution.

$$3x^2-38xy-13y^2$$
$$=3x^2-39xy+xy-13y^2$$
$$=3x(x-13y)+y(x-13y)$$
$$=(x-13y)(3x+y)$$

(vii). $5x^2+33xy-14y^2$

Solution.

$$5x^2+33xy-14y^2$$
$$=5x^2+35xy-2xy-14y^2$$
$$=5x(x+7y)-2y(x+7y)$$
$$=(x+7y)(5x-2y)$$

(viii). $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4, x\neq 0.$

Solution.

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$$

Suppose that $5x-\frac{1}{x}=y$

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4=y^2+4y+4$$
$$=y^2+2y+2y+4$$
$$=y(y+2)+2(y+2)$$
$$=(y+2)(y+2)$$
$$=(y+2)^2$$
$$=\left(5x-\frac{1}{x}+2\right)^2$$

Question.4

(i). $(x^2+5x+4)(x^2+5x+6)-3$

Solution.

$$(x^2+5x+4)(x^2+5x+6)-3$$

Suppose that $x^2+5x=y$

$$=(y+4)(y+6)-3$$
$$=y^2+6y+4y+24-3$$
$$=y^2+10y+21$$
$$=y^2+7y+3y+21$$
$$=y(y+7)+3(y+7)$$
$$=(y+7)(y+3)$$
$$=(x^2+5x+7)(x^2+5x+3)$$

(ii). $(x^2-4x)(x^2-4x-1)-20$

Solution.

$$(x^2-4x)(x^2-4x-1)-20$$

Suppose that $x^2-4x=y$

$$=(y)(y-1)-20$$
$$=y^2-y-20$$
$$=y^2-5y+4y-20$$
$$=y(y-5)+4(y-5)$$
$$=(y-5)(y+4)$$
$$=(x^2-4x-5)(x^2-4x+4)$$
$$=(x^2-5x+x-5)(x^2-2x-2x+4)$$
$$=[x(x-5)+1(x-5)][x(x-2)-2(x-2)]$$
$$=(x-5)(x+1)(x-2)(x-2)$$
$$=(x-5)(x+1)(x-2)^2$$

(iii). $(x+2)(x+3)(x+4)(x+5)-15$

Solution.

$$(x+2)(x+3)(x+4)(x+5)-15$$
$$=(x+2)(x+5)(x+3)(x+4)-15$$
$$=(x^2+5x+2x+10)(x^2+4x+3x+12)-15$$
$$=(x^2+7x+10)(x^2+7x+12)-15$$

Suppose that $x^2+7x=y$

$$=(y+10)(y+12)-15$$
$$=y^2+12y+10y+120-15$$
$$=y^2+22y+105$$
$$=y^2+15y+7y+105$$
$$=y(y+15)+7(y+15)$$
$$=(y+15)(y+7)$$
$$(x^2+7x+15)(x^2+7x+7)$$

(iv). $(x+4)(x-5)(x+6)(x-7)-504$

Solution.

$$(x+4)(x-5)(x+6)(x-7)-504$$
$$(x+2)(x+5)(x+3)(x+4)-1$$
$$=(x+4)(x-5)(x+6)(x-7)-504$$
$$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$$
$$=(x^2-x-20)(x^2-x-42)-504$$

Suppose that $x^2-x=y$

$$=(y-20)(y-42)-504$$
$$=y^2-42y-20y+840-504$$
$$=y^2-62y+336$$

$$= y^2 - 56y - 6y + 336$$
$$= y(y-56) - 6(y-56)$$
$$= (y-56)(y-6)$$
$$= (x^2 - x - 56)(x^2 - x - 6)$$
$$= (x^2 - 8x + 7x - 56)(x^2 - 3x + 2x - 6)$$
$$[x(x-8) + 7(x-9)][x(x-3) + 2(x-3)]$$
$$= (x-9)(x+7)(x-3)(x+2)$$

(v). $(x+1)(x+2)(x+3)(x+6) - 3x^2$

Solution.

$$(x+1)(x+2)(x+3)(x+6) - 3x^2$$
$$= (x+1)(x+6)(x+2)(x+3) - 3x^2$$
$$= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2$$
$$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$$
$$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$$

Suppose that $x^2 + 6 = y$

$$= (y + 7x)(y + 5x) - 3x^2$$
$$= y^2 + 5xy + 7xy + 35x^2 - 3x^2$$
$$= y^2 + 12xy + 32x^2$$
$$= y^2 + 8xy + 4xy + 32x^2$$
$$= y(y + 8x) + 4x(y + 8x)$$
$$= (y + 8x)(y + 4x)$$
$$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$$
$$= (x^2 + 8x + 6)(x^2 + 4x + 6)$$

Question.5.

(i). $x^3 + 48x - 12x^2 - 64$

Solution.

$$x^3 + 48x - 12x^2 - 64$$
$$= x^3 - 12x^2 + 48x - 64$$
$$= (x)^3 - 3(x)^2(4) + 3(x)(4)^2 - (4)^3$$
$$= (x-4)^3$$

(ii). $8x^3 + 60x^2 + 150x + 125$

Solution.

$$8x^3 + 60x^2 + 150x + 125$$
$$= 8x^3 + 60x^2 + 150x + 5^3$$
$$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$$
$$= (2x+5)^3$$

(iii). $x^3 - 18x^2 + 108x - 216$

Solution.

$$x^3 - 18x^2 + 108x - 216$$
$$= x^3 - 18x^2 + 108x - 6^3$$
$$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3$$
$$= (x-6)^3$$

(iv). $8x^3 - 125y^3 - 60x^2y + 150xy^2$

Solution.

$$8x^3 - 125y^3 - 60x^2y + 150xy^2$$
$$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$$
$$= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$$
$$= (2x - 5y)^3$$

Question.6.

(i). $27 + 8x^3$

Solution.

$$27 + 8x^3 = (3)^3 + (2x)^3$$
$$= (3 + 2x)[(3)^2 - (3)(2x) + (2x)^2]$$
$$= (3 + 2x)(9 - 6x + 4x^2)$$
$$= (2x + 3)(4x^2 - 6x + 9)$$

(ii). $125x^3 - 216y^3$

Solution.

$$125x^3 - 216y^3 = (5x)^3 - (6y)^3$$
$$= (5x - 6y)[(5x)^2 + (5x)(6y) + (6y)^2]$$
$$= (5x - 6y)(25x^2 + 30xy + 36y^2)$$

(iii). $64x^3 + 27y^3$

Solution.

$$64x^3 + 27y^3 = (4x)^3 + (3y)^3$$
$$= (4x + 3y)[(4x)^2 - (4x)(3y) + (3y)^2]$$
$$= (4x + 3y)(16x^2 - 12xy + 9y^2)$$

(iv). $8x^3 + 125y^3$

Solution.

$$8x^3 + 125y^3 = (2x)^3 + (5y)^3$$
$$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$$
$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$

Remainder Theorem and Factor Theorem:

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x - a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x = a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x = a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x - a)$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.

Exercise 5.3

Question.1. Use the remainder theorem to find the remainder when

(i). $\boldsymbol{3x^3 - 10x^2 + 13x - 6}$ ***is divided by*** $\boldsymbol{(x-2)}$

Solution.

Suppose that $\boldsymbol{p(x) = 3x^3 - 10x^2 + 13x - 6}$ ***and***

$$\boldsymbol{x - 2 = 0}$$
$$\boldsymbol{x = 2}$$

Then

$$Remainder = p(2)$$
$$= 3(2)^3 - 10(2)^2 + 13(2) - 6$$
$$Remainder = 3(8) - 10(4) + 26 - 6$$
$$= 24 - 40 + 20$$
$$= 44 - 40$$
$$= 4$$

Hence required Remainder is 4.

(ii). $4x^3 - 4x + 3$ *is divided by* $(2x - 1)$

Solution.

Suppose that $p(x) = 4x^3 - 4x + 3$ *and*

$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

Then

$$Remainder = p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$
$$Remainder = 4\left(\frac{1}{8}\right) - 2 + 3$$
$$= \frac{1}{2} + 1$$
$$= \frac{1+2}{2}$$
$$= \frac{3}{2}$$

Hence required Remainder is $\frac{3}{2}$.

(iii). $6x^4 + 2x^3 - x + 2$ *is divided by* $(x + 2)$

Solution.

Suppose that $p(x) = 6x^4 + 2x^3 - x + 2$ *and*

$$x + 2 = 0$$
$$x = -2$$

Then

$$Remainder = p(-2)$$
$$= 6(-2)^4 + 2(-2)^3 - (-2) + 2$$
$$Remainder = 6(16) + 2(-8) + 2 + 2$$
$$= 96 - 16 + 4$$
$$= 100 - 16$$
$$= 84$$

Hence required Remainder is 84.

(iv). $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ *is divided by* $(2x + 1)$

Solution.

Suppose that $p(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$ *and*

$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

Then

$$Remainder = p\left(-\frac{1}{2}\right)$$
$$= \left(2\left(-\frac{1}{2}\right) - 1\right)^3 + 6\left(3 + 4\left(-\frac{1}{2}\right)\right)^2 - 10$$
$$Remainder = (-1 - 1)^3 + 6(3 - 2)^2 - 10$$
$$= (-2)^3 + 6(1)^2 - 10$$
$$= -8 + 6 - 10$$
$$= 6 - 18$$
$$= -12$$

Hence required Remainder is -12.

(v). $x^3 - 3x^2 + 4x - 14$ *is divided by* $(x + 2)$

Solution.

Suppose that $p(x) = x^3 - 3x^2 + 4x - 14$ *and*

$$x + 2 = 0$$
$$x = -2$$

Then

$$Remainder = p(-2)$$
$$= (-2)^3 - 3(-2)^2 + 4(-2) - 14$$
$$Remainder = -8 - 3(4) - 8 - 14$$
$$= -8 - 12 - 8 - 14$$
$$= -42$$

Hence required Remainder is -42.

Question.2.

(i). If $(x + 2)$ is a factor of $x^2 - 4kx - 4k^2$, then find the value(s) of k.

Solution.

Suppose

$$p(x) = 3x^2 - 4kx - 4k^2$$

And $$x + 2 = 0$$
$$x = -2.$$

Since $(x + 2)$ is factor of the polynomial $p(x)$, So

$$p(a) = 0$$
$$3(-2)^2 - 4k(-2) - 4k^2 = 0$$
$$3(4) + 8k - 4k^2 = 0$$
$$12 + 8k - 4k^2 = 0$$
$$4k^2 - 8k - 12 = 0$$
$$4(k^2 - 2k - 3) = 0$$
$$k^2 - 2k - 3 = 0$$
$$k^2 - 3k + k - 3 = 0$$
$$k(k - 3) + 1(k - 3) = 0$$
$$(k - 3)(k + 1) = 0$$
$$k - 3 = 0, \quad k + 1 = 0$$
$$k = 3, \quad k = -1.$$

(ii). If $(x - 1)$ is a factor of $x^3 - kx^2 + 11x - 6$, then find the value(s) of k.

Solution.

Suppose

$$p(x) = x^3 - kx^2 + 11x - 6$$

And $x - 1 = 0$

$$x = 1.$$

Since $(x-1)$ is factor of the polynomial $p(x)$, So

$$p(1) = 0$$
$$(1)^3 - k(1)^2 + 11(1) - 6 = 0$$
$$1 - k + 11 - 6 = 0$$
$$6 - k = 0$$
$$k = 6.$$

Question.3.

Without actual long division determine whether (i). $(x-2)$ and $(x-3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$.

Solution.

Suppose that

$$p(x) = x^3 - 12x^2 + 44x - 48$$

And $x - 2 = 0$, $\quad x - 3 = 0$

$x = 2$, $\quad x = 3$

Remainder for $x - 2$ is

$$p(2) = (2)^3 - 12(2)^2 + 44(2) - 48$$
$$= 8 - 12(4) + 88 - 48$$
$$= 8 - 48 + 88 - 48$$
$$= 96 - 96$$
$$= 0$$

Hence $(x-2)$ *is the factor of* $p(x)$.

Remainder for $x - 3$ is

$$p(3) = (3)^3 - 12(3)^2 + 44(3) - 48$$
$$= 27 - 12(9) + 132 - 48$$
$$= 27 - 108 + 132 - 48$$
$$= 159 - 156$$
$$= 3 \neq 0$$

Hence $(x-3)$ *is not factor of* $p(x)$.

Question.4.

For what value of m is the polynomial $p(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $(x+2)$?

Solution.

Suppose that

$$p(x) = 4x^3 - 7x^2 + 6x - 3m$$

And $x + 2 = 0$

$$x = -2.$$

Remainder for $x + 2$ is

$$p(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$$
$$= 4(-8) - 7(4) - 12 - 3m$$
$$= -32 - 28 - 12 - 3m$$
$$= -72 - 3m$$

For the given condition $p(-2) = 0$

$$-72 - 3m = 0$$
$$-3m = 72$$
$$m = -\frac{72}{3} = -24.$$

Question.5.

Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution.

Let $p(x) = kx^3 + 4x^2 + 3x - 4.$

By remainder theorem, $p(x)$ is divided by $(x-3)$, then remainder is

$$p(3) = k(3)^3 + 4(3)^2 + 3(3) - 4$$
$$p(3) = k(27) + 4(9) + 9 - 4$$
$$p(3) = 27k + 36 + 5$$
$$p(3) = 27k + 41$$

Also by remainder theorem, $q(x)$ is divided by $(x-3)$, then remainder is

$$q(3) = (3)^3 - 4(3) + k$$
$$q(3) = 27 - 12 + k$$
$$q(3) = 15 + k$$

By Given Condition, we have

$$p(3) = q(3)$$
$$27k + 41 = 15 + k$$
$$27k - k = 15 - 41$$
$$26k = -26$$
$$k = -1.$$

Question.6.

The remainder after dividing the polynomial $p(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$. Calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being divided by $(x-2)$.

Solution.

Let $p(x) = x^3 + ax^2 + 7$ by $(x+1)$.

By remainder theorem, $p(x)$ is divided by $(x+1)$, then remainder is 2b

$$p(-1) = 2b$$
$$(-1)^3 + a(-1)^2 + 7 = 2b$$
$$-1 + a + 7 = 2b$$
$$a + 6 = 2b$$
$$a = 2b - 6 - - - (i)$$

By remainder theorem, $p(x)$ is divided by $(x-2)$, then remainder is $b + 5$

$$p(2) = b + 5$$
$$(2)^3 + a(2)^2 + 7 = b + 5$$
$$8 + a(4) + 7 = b + 5$$
$$4a + 15 = b + 5$$
$$4a = b - 10 - - - (ii)$$

Using (i) in (ii), we have

$$4(2b - 6) = b - 10$$
$$8b - 24 = b - 10$$

$$8b - b = -10 + 24$$
$$7b = 14$$
$$b = \frac{14}{7}$$
$$b = 2$$

Using $b = 2\ in\ (ii), we\ have$

$$4a = 2 - 10$$
$$4a = -8$$
$$a = -\frac{8}{4}$$
$$a = -2$$

Hence $a = -2\ and\ b = 2.$

Question.7.

The polynomial $\boldsymbol{x^3 + lx^2 + mx + 24}$ has a factor $(\boldsymbol{x + 4})$ and it leaves a remainder of $\boldsymbol{36}$ when divided by $(\boldsymbol{x - 2})$. Find the values of $\boldsymbol{l\ and\ m}$.

Solution.

Let $p(x) = x^3 + lx^2 + mx + 24.$

By remainder theorem, $p(x)$ is divided by $(x + 4)$, then remainder is 0.

$$p(-4) = 0$$
$$(-4)^3 + l(-4)^2 + m(-4) + 24 = 0$$
$$-64 + 16l - 4m + 24 = 0$$
$$16l - 4m - 40 = 0$$
$$4(4l - m - 10) = 0$$
$$4l - m - 10 = 0$$
$$4l - m = 10 - - - (i).$$

By remainder theorem, $p(x)$ is divided by $(x - 2)$, then remainder is 36.

$$p(2) = 36$$
$$(2)^3 + l(2)^2 + m(2) + 24 = 36$$
$$8 + 4l + 2m + 24 = 36$$
$$4l + 2m + 32 = 36$$
$$4l + 2m = 36 - 32$$
$$4l + 2m = 4$$
$$2(2l + m) = 4$$
$$2l + m = \frac{4}{2}$$
$$2l + m = 2 - - - (ii)$$

Using (i) in (ii), we have

$$4l - m = 10$$
$$2l + m = 2$$
$$\overline{\quad 6l = 12 \quad}$$
$$l = \frac{12}{6}$$
$$l = 2.$$

Using $l = 2\ in\ (ii), we\ have$

$$2(2) + m = 2$$
$$4 + m = 2$$
$$m = 2 - 4$$
$$m = -2.$$

Hence $l = 2\ and\ m = -2.$

Question.8.

The expression $\boldsymbol{lx^3 + mx^2 - 4\ leaves}$ remainder $\boldsymbol{-3\ and\ 12\ respectively}$ when divided by the $(\boldsymbol{x - 1})\boldsymbol{and}\ (\boldsymbol{x + 2})$ respectively. Calculate the values of $\boldsymbol{l\ and\ m}$.

Solution.

Let $p(x) = lx^3 + mx^2 - 4.$

By remainder theorem, $p(x)$ is divided by $(x - 1)$, then remainder is 0.

$$p(1) = -3$$
$$l(1)^3 + m(1)^2 - 4 = -3$$
$$l + m = -3 + 4$$
$$l + m = 1$$
$$l = 1 - m - - - (i)$$

By remainder theorem, $p(x)$ is divided by $(x + 2)$, then remainder is 12.

$$p(-2) = 12$$
$$l(-2)^3 + m(-2)^2 - 4 = 12$$
$$-8l + 4m - 4 = 12$$
$$-8l + 4m = 12 + 4$$
$$-8l + 4m = 16$$
$$4(-2l + m) = 16$$
$$-2l + m = \frac{16}{4}$$
$$-2l + m = 4 - - - (ii)$$

Using (i) in (ii) , we have

$$-2(1 - m) + m = 4$$
$$-2 + 2m + m = 4$$
$$3m = 4 + 2$$
$$3m = 6$$
$$m = \frac{6}{3}$$
$$m = 2.$$

Using value of m in (ii), we have

$$-2l + 2 = 4$$
$$-2l = 4 - 2$$
$$-2l = 2$$
$$l = -\frac{2}{2}$$
$$l = -1.$$

Hence $l = -1\ and\ m = 2.$

Question.9.

The expression $\boldsymbol{ax^3 - 9x^2 + bx + 3a}$ is exactly divisible by $\boldsymbol{x^2 - 5x + 6}$. Find the values of $\boldsymbol{a\ and\ b}$.

Solution.

Let $p(x) = ax^3 - 9x^2 + bx + 3a$

As

$$x^2 - 5x + 6 = 0$$
$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-3)(x-2) = 0$$
$$x - 3 = 0\,, x - 2 = 0$$

By remainder theorem, $p(x)$ is divided by $(x-3)$, then remainder is 0.

$$p(3) = 0$$
$$a(3)^3 - 9(3)^2 + b(3) + 3a = 0$$
$$27a - 81 + 3b + 3a = 0$$
$$30a + 3b = 81 --- (i)$$

By remainder theorem, $p(x)$ is divided by $(x-2)$, then remainder is 0.

$$p(2) = 0$$
$$a(2)^3 - 9(2)^2 + b(2) + 3a = 0$$
$$8a - 36 + 2b + 3a = 0$$
$$11a + 2b = 36 --- (ii)$$

Multiply equation (i) by 2 and equation (ii) by 3, then subtracting eq. (ii) from (i), we have

$$60a + 6b = 162$$
$$\pm 33a \pm 6b = \pm 108$$
$$\overline{\quad 27a = 54 \quad}$$
$$a = 2$$

Put $a = 2\ in\ eq. (i), we\ get$

$$30(2) + 3b = 81$$
$$60 + 3b = 81$$
$$3b = 81 - 60$$
$$3b = 21$$
$$b = 7$$

Hence $a = 2\ and\ b = 7.$

Rational Root Theorem

Let

$a_0x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1}x + a_n, \quad a_n \neq 0 \ldots\ldots$ (i)

Be a polynomial equation of degree n with integral coefficients . If $\frac{p}{q}$ is a rational root of the equation, then p is a factor of the constant term a_n and q is the factor of *leading coefficient* a_0.

EXERCISE# 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q#1) $\boldsymbol{x^3 - 2x^2 - x + 2}$

Solution: Let $P(x) = x^3 - 2x^2 - x + 2 \ldots(1)$

Here, the constant term is 2 and factors of constant terms are $\pm 1, \pm 2, \pm 4, \ldots$

Therefore, we check $\pm 1, \pm 2, \pm 4$ for the roots.

Now, put $x = 1$ in (1), we have

$P(1) = (1)^3 - 2(1)^2 - (1) + 2$

$P(1) = 1 - 2 - 1 + 2$

$P(1) = 0$

Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.

Now, put $x = 2$ in (1), we have

$P(2) = (2)^3 - 2(2)^2 - (2) + 2$

$P(2) = 8 - 8 - 2 + 2$

$P(2) = 0$

Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.

Now, put $x = -1$ in (1), we have

$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$P(-1) = -1 - 2 + 1 + 2$

$P(-1) = 0$

Hence, $x = -1$ is the root of $P(x)$, therefore $\big(x - (-1)\big) = (x + 1)$ is the factor of $P(x)$.

Thus, $P(x) = (x-1)(x-2)(x+1)$

Q#2) $\boldsymbol{x^3 - x^2 - 22x + 40}$

Solution: Let $P(x) = x^3 - x^2 - 22x + 40 \ldots(1)$

Here, the constant term is 40 and factors of constant terms are $\pm 1, \pm 2, \pm 4, \pm 5 \ldots$

Therefore, we check $\pm 1, \pm 2, \pm 4, \pm 5$ for the roots.

Now, put $x = 1$ in (1), we have

$P(1) = (1)^3 - (1)^2 - 22(1) + 40$

$P(1) = 1 - 1 - 22 + 40$

$P(1) = 18 \neq 0$

Hence, $x = 1$ is the not the root of $P(x)$,

Now, put $x = 2$ in (1), we have

$P(2) = (2)^3 - (2)^2 - 22(2) + 40$

$P(2) = 8 - 4 - 44 + 40$

$$P(2) = 48 - 48 = 0$$

Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = 4$ in (1), we have
$P(4) = (4)^3 - (4)^2 - 22(4) + 40$
$P(4) = 64 - 16 - 88 + 40$
$P(4) = 104 - 104 = 0$
Hence, $x = 4$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -5$ in (1), we have
$P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$
$P(-5) = -125 - 25 + 110 + 40$
$P(-5) = 150 - 150 = 0$
Hence, $x = -5$ is the root of $P(x)$, therefore $\big(x - (-5)\big) = (x + 5)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x - 4)(x + 5)$

Q#3) $\boldsymbol{x^3 - 6x^2 + 3x + 10}$

Solution: Let $P(x) = x^3 - 6x^2 + 3x + 10 \ldots(1)$
Here, the constant term is 10 and factors of constant terms are $\pm1, \pm2, \pm4, \pm5$ ...
Therefore, we check $\pm1, \pm2, \pm4, \pm5$ for the roots.
Now, put $x = -1$ in (1), we have
$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$
$P(-1) = -1 - 6 - 3 + 10$
$P(-1) = 0$
Hence, $x = -1$ is the root of $P(x)$, therefore $(x — 1) = (x + 1)$ is the factor of $P(x)$.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10$
$P(2) = 8 - 24 + 6 + 10$
$P(2) = 24 - 24 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = 5$ in (1), we have
$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10$
$P(5) = 125 - 6(25) + 15 + 10$
$P(5) = 125 - 150 + 25 = 0$
Hence, $x = 5$ is the root of $P(x)$, therefore $(x - 5)$ is the factor of $P(x)$.
Thus, $P(x) = (x + 1)(x - 2)(x - 5)$

Q#4) $\boldsymbol{x^3 + x^2 - 10x + 8}$

Solution: Let $P(x) = x^3 + x^2 - 10x + 8 \ldots(1)$
Here, the constant term is 8 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$
Therefore, we check $\pm1, \pm2, \pm4$ for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = (1)^3 + (1)^2 - 10(1) + 8$
$P(1) = 1 + 1 - 10 + 8$
$P(1) = 10 - 10 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 + (2)^2 - 10(2) + 8$
$P(2) = 8 + 4 - 20 + 8$
$P(2) = 20 - 20$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is also a factor of $P(x)$.
Now, put $x = -4$ in (1), we have
$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$
$P(-4) = -64 + 16 + 40 + 8$
$P(-4) = -64 + 64 = 0$
Hence, $x = -4$ is the root of $P(x)$, therefore $\big(x - (-4)\big) = (x + 4)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x - 2)(x + 4)$

Q#5) $\boldsymbol{x^3 - 2x^2 - 5x + 6}$

Solution: Let $P(x) = x^3 - 2x^2 - 5x + 6 \ldots(1)$
Here, the constant term is 2 and factors of constant terms are $\pm1, \pm2, \pm3, \ldots$
Therefore, we check $\pm1, \pm2, \pm3$ for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$
$P(1) = 1 - 2 - 5 + 6$
$P(1) = 7 - 7 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = 3$ in (1), we have
$P(3) = (3)^3 - 2(3)^2 - 5(3) + 6$
$P(2) = 27 - 2(9) - 15 + 6$
$P(2) = 27 - 18 - 15 + 6 = 33 - 33 = 0$
Hence, $x = 3$ is the root of $P(x)$, therefore $(x - 3)$ is the factor of $P(x)$.
Now, put $x = -2$ in (1), we have
$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$
$P(-2) = -8 - 2(4) + 10 + 6$
$P(-2) = -8 - 8 + 10 + 6 = -16 + 16 = 0$
Hence, $x = -2$ is the root of $P(x)$, therefore $\big(x - (-2)\big) = (x + 2)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x - 3)(x + 2)$

Q#6) $\boldsymbol{x^3 + 5x^2 - 2x - 24}$

Solution: Let $P(x) = x^3 + 5x^2 - 2x - 24 \ldots(1)$
Here, the constant term is 24 and factors of constant terms are $\pm1, \pm2, \pm4, \ldots$
Therefore, we check $\pm1, \pm2, \pm4$ for the roots.
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$
$P(2) = 8 + 5(4) - 4 - 24$
$P(2) = 8 + 20 - 4 - 24 = 28 - 28 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = -3$ in (1), we have
$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$
$P(-3) = -27 + 5(9) + 6 - 24$

$P(-3) = -27 + 45 + 6 - 24 = 51 - 51 = 0$
Hence, $x = -3$ is the root of $P(x)$, therefore $(x + 3)$ is the factor of $P(x)$.
Now, put $x = -4$ in (1), we have
$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$
$P(-4) = -64 + 5(16) + 8 - 24$
$P(-4) = -64 + 80 + 8 - 24 = 88 - 88 = 0$
Hence, $x = -4$ is the root of $P(x)$, therefore $\big(x - (-4)\big) = (x + 4)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x + 3)(x + 4)$

Q#7) $3x^3 - x^2 - 12x + 4$

Solution: Let $P(x) = 3x^3 - x^2 - 12x + 4 \ldots(1)$
Here, the constant term is 4 and factors of constant terms are $\pm 1, \pm 2, \pm 4, \ldots$
Therefore, we check $\pm 1, \pm 2, \pm 4$ for the roots.
Now, put $x = 2$ in (1), we have
$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$
$P(2) = 3(8) - 4 - 24 + 4$
$P(2) = 24 - 4 - 24 + 4 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = -2$ in (1), we have
$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$
$P(-2) = 3(-8) - 4 + 24 + 4$
$P(-2) = -24 - 4 + 24 + 4 = 0$
Hence, $x = -2$ is the root of $P(x)$, therefore $\big(x - (-2)\big) = (x + 2)$ is the factor of $P(x)$.
Since the leading co-efficient is 3, therefore we also check at $x = \frac{1}{3}$ and $x = -\frac{1}{3}$
First we check at $x = \frac{1}{3}$

$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4$$
$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{27}\right) - \left(\frac{1}{9}\right) - 4 + 4$$
$$P\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{9} - 4 + 4 = 0$$

Hence, $x = \frac{1}{3}$ is the root of $P(x)$, therefore $\left(x - \frac{1}{3}\right) = 0$ gives that $(3x - 1)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x + 2)(3x - 1)$

Q#8) $2x^3 + x^2 - 2x - 1$

Solution: Let $P(x) = 2x^3 + x^2 - 2x - 1 \ldots(1)$
Here, the constant term is 1 and factors of constant terms are ± 1 .
Therefore, we check ± 1 for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$
$P(1) = 2 + 1 - 2 - 1$
$P(1) = 3 - 3 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -1$ in (1), we have
$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$P(-1) = -2 + 1 + 2 - 1$
$P(-1) = -3 + 3 = 0$
Hence, $x = -1$ is the root of $P(x)$, therefore $\big(x - (-1)\big) = (x + 1)$ is the factor of $P(x)$.
Since the leading co-efficient is 2, therefore we check $x = -\frac{1}{2}$ and $x = \frac{1}{2}$
First we check at $x = -\frac{1}{2}$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1$$
$$P\left(-\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - \left(\frac{1}{4}\right) + 1 - 1$$
$$P\left(-\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} + 1 - 1 = 0$$

Hence, $x = -\frac{1}{2}$ is the root of $P(x)$, therefore $\left(x + \frac{1}{2}\right) = 0$ gives that $(2x + 1)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x + 1)(2x + 1)$